Multi-scale geometric Ssegmentation of uniform regions in multi-spectral images

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Abstract

Segmentation in general and segmentation of multi-spectral images in particular require advances analysis and computational methodologies. In this paper we introduce a new state of the art method for segmentation of multi-spectral images. The proposed methodology is based on a multi-scale geometric transformation called the Beamlet Transform. The method is applicable for both mono-spectral and multi-spectral images where each pixel has its corresponding spectral profile vector. The proposed segmentation method is especially effective when the underlying image consist of relatively large segment with smooth boundaries, in this case it perform exceptional well even in extremely low SNR. The method is unsupervised and assume no prior knowledge of the image characteristics or features.
In order to validate the efficiency of our method we used the known Lark algorithm as a benchmark for segmentation of multi-spectral images and show that our new method out-performs the Lark algorithm.

1. Introduction and background

The segmentation problem involves the decomposition of an image into a set of disjoint segments or blocks where each block is homogeneous with respect to its interior and heterogeneous with respect to its neighbor segments. In this work we present a state of the art method for the segmentation of digital images from different dimensions and kinds that is based on the principles of multi-scale geometric analysis. In order to understand the segmentation method and its building blocks we will first review the related geometrical framework and transformation and in particular Beamlets and the Beamlet Transform.

1.1 Beamlets and the Beamlet Transform

Beamlets were first introduced by Donoho and Huo [Donoho and Huo 2001]. The Beamlet Transform is actually an extention of the known Radon Transform [Dyn 2000] over a multi-scale set of line-segments. The Discrete Radon Transform computes a set of line integral over a set of global lines over uniformly sampled sets of orientations and locations with respect to the image [See Figure 1]

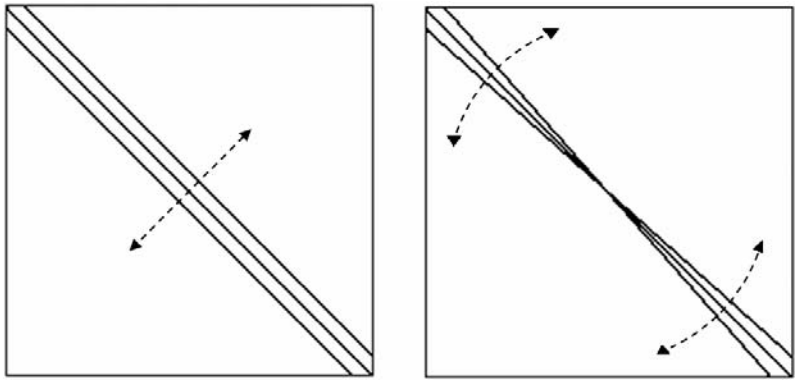

Figure 1 - Uniform sampling of orientations and locations of lines and their intersections with the image

The 2-dimensional Beamlet Transform applies the standard Radon Transform over a hierarchical multi-scale structure of regions in the image known as the quad tree split of the image. The first level of the quad tree consist of the entire image, the second level contains the 4 quadrants of the image, the 3rd level contains the division of the image into 16 disjoint squares and so on as shown in Figure 2. The finest level of the quad tree can contain the individual image pixel or larger squares, as desired. The Beamlet set consist of line segments that connects points on the boundaries of the dyadic squares in the quad tree.

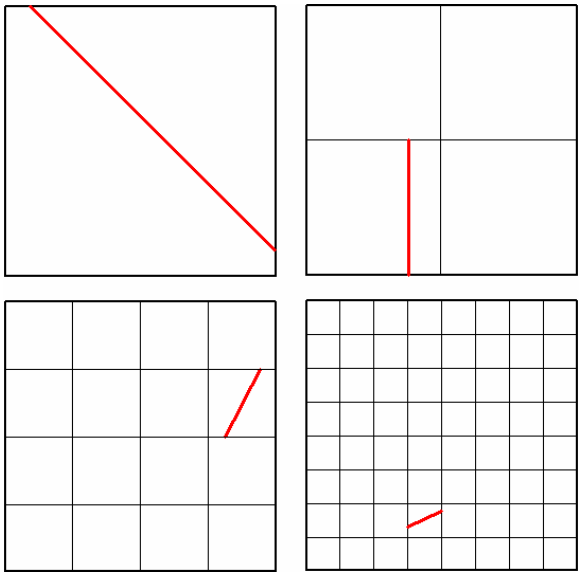

Figure 2 – The quad tree spilt of an image and corresponding bamlets.

It can be easily shown that if we connect every disjoint pair of boundary grid points for every dyadic square in the quad tree defined over an n by n image we get a total of $o(\log n \cdot n^2)$ line segment compare to the set of $o(n^4)$ segments connecting every pair of grid points in the digital image. Despite of being compact, the beamlet set make it possible to approximate any line segment in the image using a connected chain of at most $4 \cdot \log(n)$ beamlets within a Hausdorff

1.2 Approximating a digital image using the standard quad tree

A quad tree approximation of a digital image is obtained by a recursive process where in each stage there are two possible decision to be made, 1 - approximating a given dyadic square as a uniform region with the average corresponding pixels value and stop, or 2 - apply a quad split and continue with the recursion on each one of the resulting squares. At the end of the process, a piece-wise constant approximation of the image is obtained where the constant regions are squares at different sizes as illustrated in Figure 3.

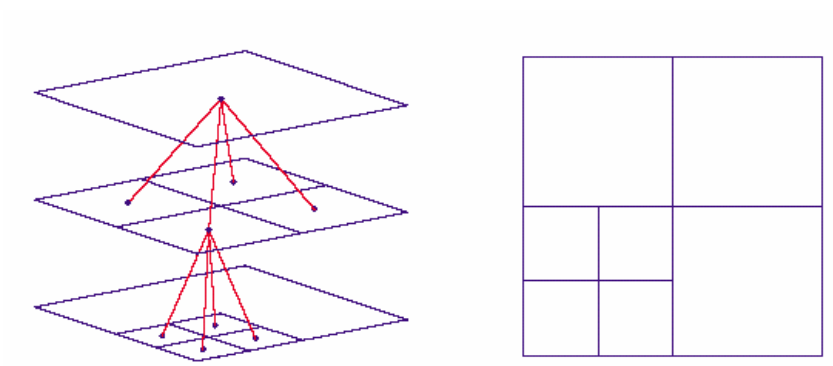

Figure 3 – A quad tree split of an image boundary.

1.3 Approximation an image using a beamlet decorated quad tree

Same as the standard quad tree approximation, the beamlet decorated one [Donoho and Hou 1999] is the result of a recursive process applied to the dyadic squares of an image but in addition to the 2 choices for the decision in each stage, a third possible decision is applying a line split using the optimal line choice over the given square and then stop as shown in Figure 4. Each region of the square after a line split is the average of the corresponding pixels set values.

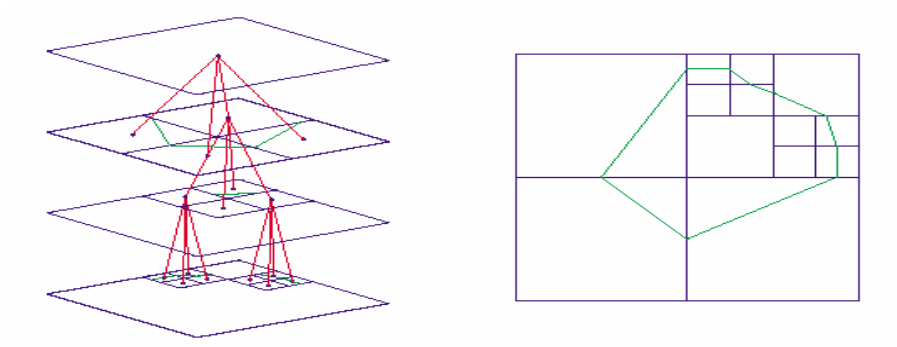

Figure 4 – A beamlet decorated split of an image boundary

The beamlet decorated quad tree also provides a piece-wise constant approximation of the image but the variety of shapes of the constant regions is much larger and more flexible which makes it possible to achieve much better approximation of the image using much fewer segments. Figure 5 show a comparison between the standard quad tree algorithm and the beamlet decorated one when applied to a simple binary image consisting of a single edge.

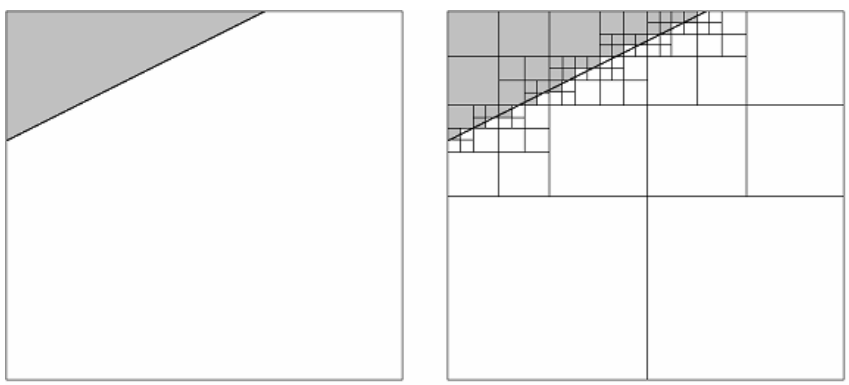

Figure 5 - comparison between the standard quad tree algorithm and the beamlet decorated one

If carried out explicitly, the operation of comparing all possible linear splits and choosing the best one should take $o(n^4)$ operations for an n by n image and make our algorithm impractical for even moderately large images. We have developed an implicit evaluation of the splits by using FFT's and smart updates between similar splits and by doing so we were able to reduce the computational time to $o(\log n \cdot n^2)$ only.

2. The method

Our segmentation method consist of 2 main phases, the split phase and the merge phase.

2.1 The split phase

The split phase consists of the following steps

1. Finding the optimal split for each one of the dyadic squares in the image all the way to the finest scale using a simple least squares criteria for the choices of splits described below.
2. At this step we find the best beamlet decorated quad tree approximation of the image by folding the tree nodes up every time were a penalized residual sum of squares measure such as BIC below can be improved**.** Each time for a give square it has to be decided whether leave the current optimal split or to fold the block up in the tree.

$$BIC = \ln(N) \cdot K + N \cdot \ln(RSS)$$

$$RSS = \sum_{k=1}^{K} \sum_{i=1}^{N_k} (y_{i,k} - \overline{y_k})^2$$

Equation 1 – BIC and RSS where N is the total number of pixel in an image and K is the number of blocks.

The output at the end of the split phase is a beamlet decorated quad tree approximation of the image like in Figure 6

Figure 6 – The original Lenna image on the left and its corresponding beamlet decorated quad tree on the right.

2.2 The merging phase

The goal in the merging phase is merging neighbor blocks so the over all statistic measure for the approximation quality (BIC for example) will be improved at each stage. Of course that it is impractical to find the optimal split of the entire image since it's an NP hard problem, instead we apply a greedy approach that optimizes locally at every step until it is impossible to improve the statistical criteria by merging a pair from the current block set.

Figure 7 illustrates a situation were it was found optimal to merge blocks M1, M4 and M5 at the merging phase, such a merge was impossible in the splitting phase since the blocks belong to different dyadic squares.

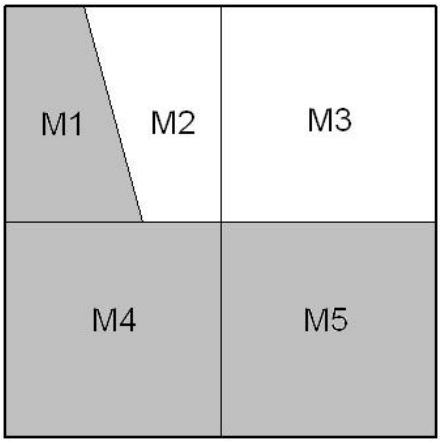

Figure 7 – The merging phase enable to merge blocks that couldn't be merged in the previous phase of the algorithm due to the quad tree structure.

Figure 8 shows the results of our algorithm when applied to an agricultural region image

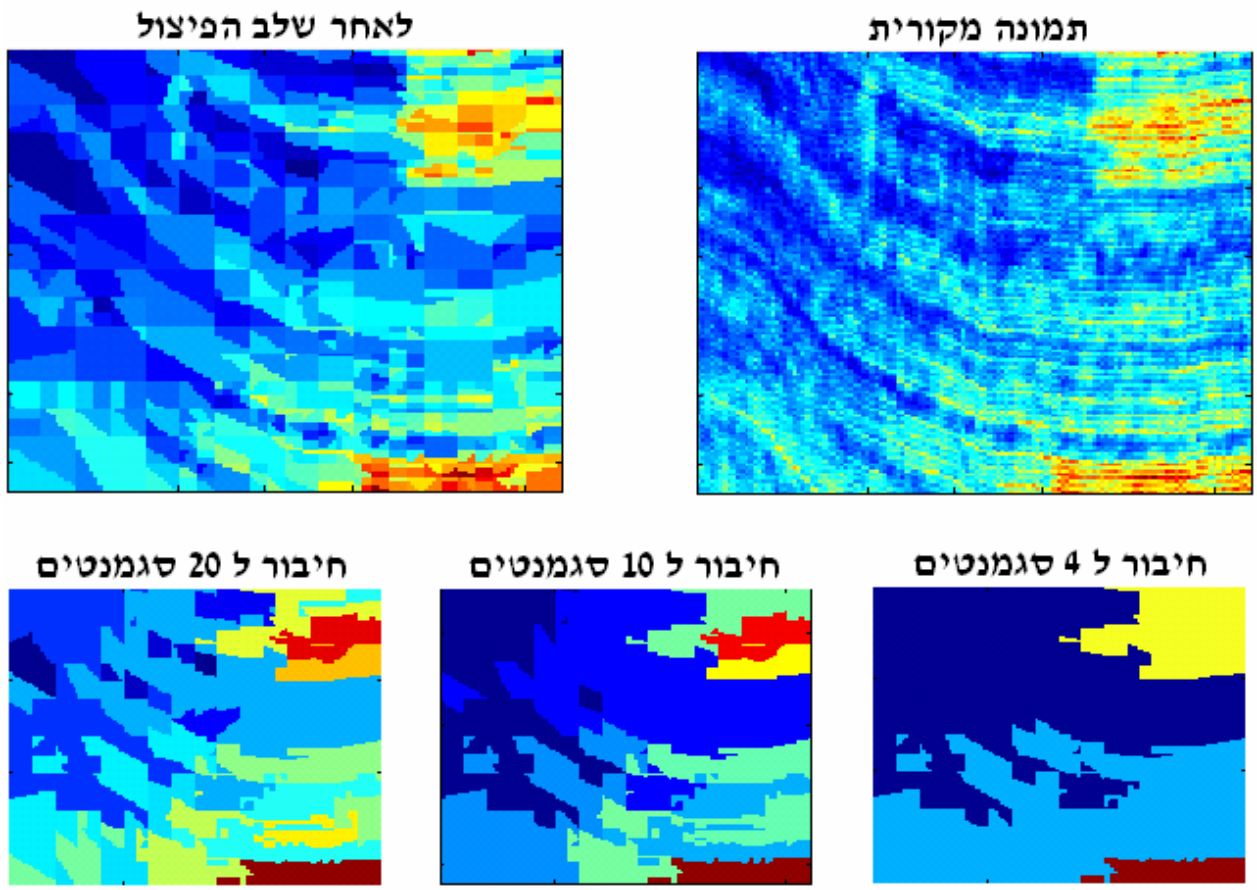

Figure 8 – Example of segmentation of a mono-spectral agricultural region image

2.3 Generalization to multi-spectral images

The chosen statistical criteria, for comparison between different approximation of an image in our previous examples was applied to the specific pixel values, however, it is straight forward to generalize it to the multi-spectral case were the residual sum of squares is computed separately for every spectral channel and them summed up and penalized by the total number of blocks using an adjustment parameter as shown in equation 2 were f is a monotonous non-decreasing function.

$$\sum_{j=1}^{N} RSS_j + \alpha \cdot f(K) \tag{2}$$

The adjustment parameter alpha is used to determine the trade of between approximation error and approximation complexity.

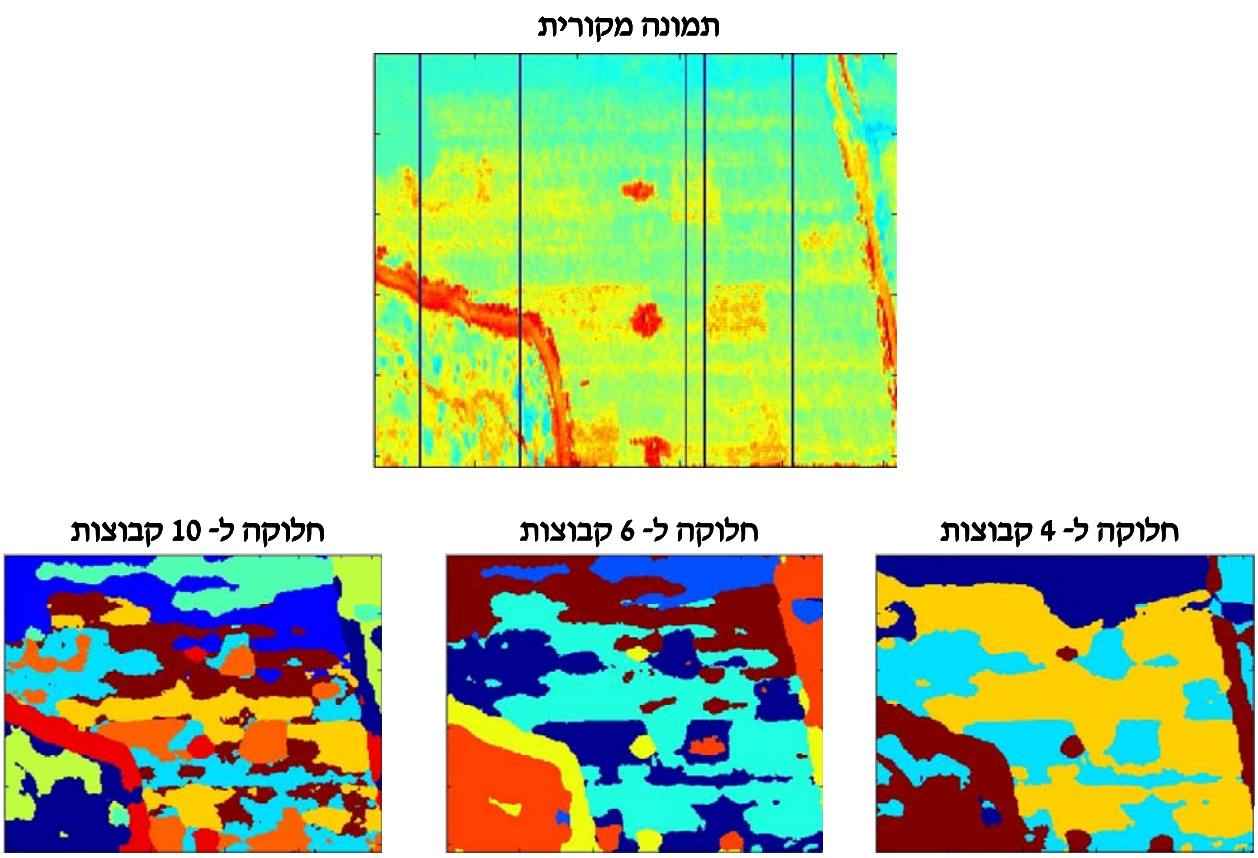

Figure 9 – Segmentation of a multi-spectral agricultural region image.

2.4 Comparison with the Lark algorithm

The one of the most widely used algorithm for segmentation of hyper spectral images is the Lark Algorithm [Lark and Stafford 98]. In Figure 10 we show a comparison between the Lark algorithm and our new proposed segmentation method for a simple multi-spectral image with very low SNR. It can be shown that the Lark algorithm failed in the segmentation were our method revealed the underlying segments. We were able to get reasonable results using Lark only when the level of noise was much lower.

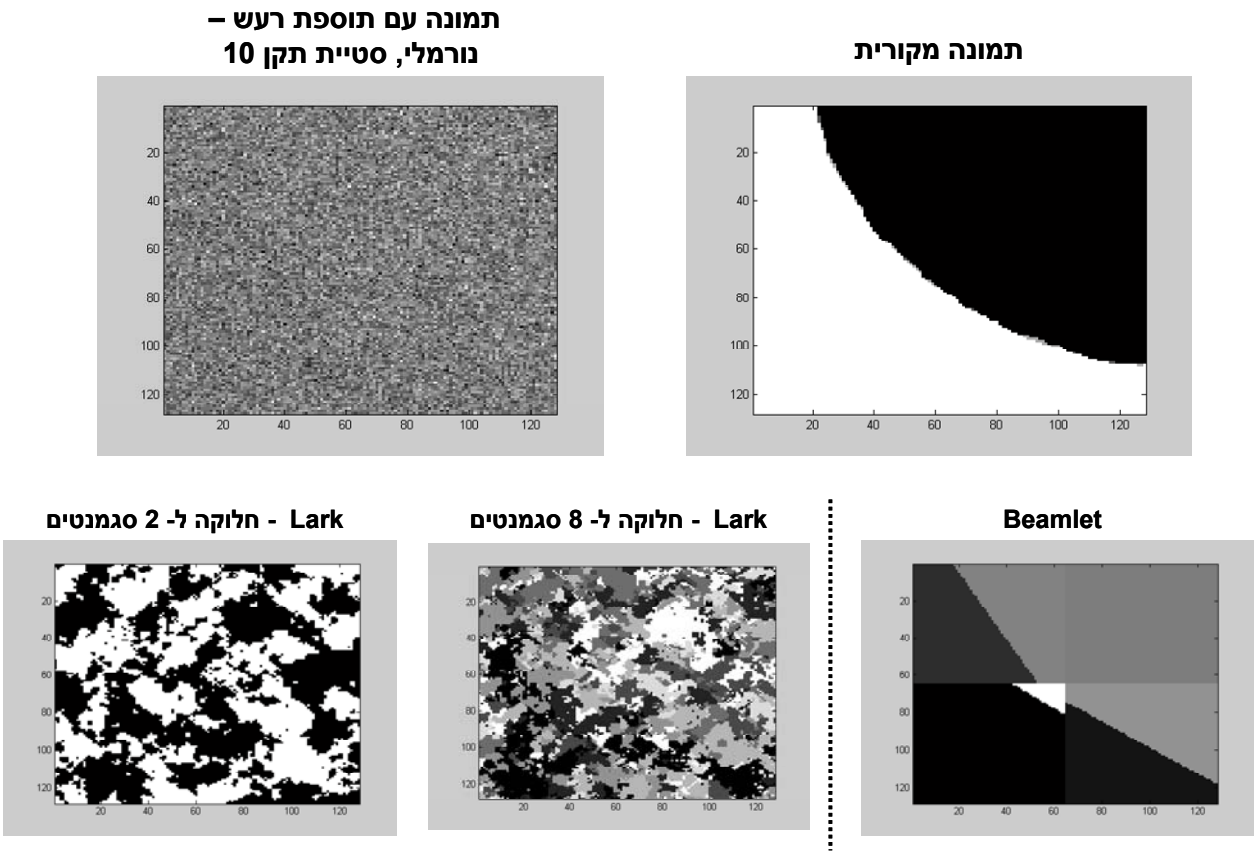

Figure 10 – Comparison between Lark and Beamlets

3. Summary and Conclusions

We have introduced a new algorithm for segmentation of multi-spectral images.our method is based on a multi-scale geometric approach combined with highly efficient computational methods that is capable to approximate well a variety of shapes and scales in an image with a We

have compared our method to the Lark algorithm and found that our method can handle much more efficiently images with very low SNR.

Refferences

David L. Donoho, Wedgelets: Nearly-Minimax Estimation of Edges, Annals of Stat., vol. 27, 1999, pp. 859-897.

David L. Donoho and Xiaoming Huo, Beamlets and Multiscale Image Analysis, Springer Lecture Notes in Computational Science and Engineering, Vol. 20, 2002, pp. 149-196.

Lark, R. M. and Stafford, J. V. (1998) "Information on within-Field Variability from Sequences of Yield Maps: Multivariate Classification as a First Step of Interpretation.", Nutrient Cycling in Agro-ecosystems, 50, 277-281

Lark, R. M. (1998) "Forming Spatially Coherent Regions by Classification of Multivariate Data.", International Journal of Geographical Information Science., 12, 83-98

Dyn (2000) The Radon Transform and it's applications.